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Using Differential Transform Method to Solve Non-Linear Heat Equation

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Abstract: **The paper presents a review of Differential Transform Method (DTM) to solve non-linear heat equations. The method reduces the large amount of computational methods when non-linear problems need to be solved. The solutions obtained are exact to the high power of derivatives. After a brief discussion of the methods available, the method was used to solve an example with two variables. Approximate and exact solutions were developed for two derivatives, and the accuracy of DTM is evident in solving non-linear heat equations.**

Keywords**: Non-linear equations, heat equation, differential transform method.**

1. INTRODUCTION

Differential Transform Method (DTM) is based in the Taylor Series and is used to solve non-linear and linear problems in heat and electrical systems. It is easy to core in solving engineering problems, and solutions are developed without detailed discretisation and linearisation. The use of DTM methods reduces the computational work, and precise solutions can be developed for the problems. Many phenomena are non-linear, and these require non-linear solutions. The advantage of DTM is that helps in calculating the kth derivatives when the boundary conditions are known and unknown (Hossein and Milad, 2010). This paper uses DTM to solve non-linear heat equations.

2. ANALYSIS OF THE METHOD

Non-linear reaction diffusions are written as per the following form. The equation is used to represent a number of heat, chemical, and other phenomena (Kangalgil and Ayaz, 2007):

$$u_t = (A(u)u_x)_x + B(u)u_x + C(u) \qquad (1)$$

Where $A(u), B(u)$ and $C(u)$ are arbitrary functions *t* and *x* are indices for differentiation $u = u(t;x)$ is the function that needs to be derived. For this paper, the following heat equation is developed from (1).

$$u_t = u_{xx} + \varepsilon u^m \qquad (2)$$

In the above equation:

ε is a parameters

t, *x* are derivatives of variables, *m = 0,1,2,* and so on.

Developing such equations for non-linear problems is important. Diffusion problem with ε = 0 is simple. When diffusion is combined with reaction kinetics, then changes are quicker than straight equations such as ε = 0. This type of combination creates reaction diffusion of the ε sum indicating the kinetics. As an example, for m = 3 in (2), a heat equation with cubic non-linearity is developed as given below (Selvi and Ramesh, 2017):

$$u_t = (A(u)u_x)_x + C(u) \qquad (3)$$

The above equation is developed for function $A(u)$ and $C(u)$ with the antireductionist method.

A two-dimensional differential transform is developed for function $w(x,y)$ for an analytic and differentiated transform with reference to time and *y* as:

$$W(k,h) = \frac{1}{k!\,h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial y^h} w(x,y)\right]_{\substack{x=x_0\\ y=y_0}} \qquad (4)$$

In the above equation, $W(k,h)$ is a spectrum that is transformed and is called the T function. The lower case of variables $w(x,y)$ are the original function, whilst the uppercase $W(k,h)$ are the transformed function. The differential inverse transform of the value is (Alhaddad, 2017):

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h)(x-x_0)^k (y-y_0)^h \qquad (5)$$

Combing equations (4) and (5) gives:

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!\,h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial y^h} w(x,y)\right]_{\substack{x=x_0\\ y=y_0}} (x-x_0)^k (y-y_0)^h \qquad (6)$$

In the above equation, *(0, 0)* is substituted for (x_0, y_0). In the above equation, when (0, 0) is substituted for (x_0, y_0), then the expression is:

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!\,h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial y^h} w(x,y)\right]_{\substack{x=x_0\\ y=y_0}} x^k y^h \qquad (7)$$

Equation (7) now becomes:

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h) x^k y^h \qquad (8)$$

The above equation implies that the value of $w(x,y)$ is very small and it can be neglected. For real applications, n and m values are determined by zeros coefficients convergence. For heat equation equation, (8) is (Jawad and Hamody, 2014):

$$w(x,y) = \sum_{k=0}^{m}\sum_{h=0}^{n} W(k,h) x^k y^h \qquad (9)$$

The above method is used to directly transform certain function. Given below are some original functions and the transformed functions (Vedat, 2007):

Functional Form	Transformed Form
$w(x,t) = u(x,t) \pm v(x,t)$	$W(k,h) = U(k,h) \pm V(k,h)$
$w(x,t) = \alpha u(x,t)$	$W(k,h) = \alpha U(k,h)$ $\quad , \alpha\ is\ a\ constant$
$w(x,t) = x^m t^n$	$W(k,h) = \delta(k-m, h-n) = \delta(k-m)\delta(h-n)$ $\delta(-m, h-n) = \begin{cases} 1, & k = m\ and\ h = n \\ 0, & otherwise \end{cases}$
$w(x,t) = u(x,t)v(x,t)$	$W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)V(k-r,s)$
$w(x,t) = \frac{\partial}{\partial x} u(x,t)$	$W(k,h) = (k+1)U(k+1,h)$
$w(x,t) = \frac{\partial}{\partial t} u(x,t)$	$W(k,h) = (h+1)U(k,h+1)$

$w(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$	$W(k,h) = \sum_{r=0}^{k} \sum_{s=0}^{h} (k-r+2)(k-r+1) U(r,h-s) V(k-r+2,s)$
$w(x,t) = u(x,t) \frac{\partial}{\partial x} u(x,t)$	$W(k,h) = \sum_{r=0}^{k} \sum_{s=0}^{h} (k-r+1) U(r,h-s) U(k-r+1,s)$

3. APPLICATIONS

This section presents some examples of non-linear equations developed using the method developed in the above section (Kangalgil and Ayaz, 2007).

For the conditions of $\boldsymbol{\varepsilon = -2}$ and $\boldsymbol{m = 3}$

The values are substituted in (2) to obtain:

$$u_t = u_{xx} - 2u^3 \qquad (10)$$

DTM is used to solve (1) with initial conditions of:

$$u(x,0) = \frac{1+2x}{x^2+x+1} \qquad (11)$$

With boundary conditions assumed as:

$$u(0,t) = \frac{1}{6t+1}\ , \qquad u_x(0,t) = \frac{12t+1}{(6t+1)^2} \qquad (12)$$

The two dimensional change of (10) is given as:

$$(h+1)U(k,h+1) = (k+1)(k+2)U(k+2,h) - 2\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(k-r-t,p)Ur,h-s-p)U(t,s) \qquad (13)$$

The equations (11) and (8), the transformational coefficients $U(k,0), k = 0,1,2,\ldots$ are:

$$U(0,0) = 1, U(1,0) = 1, U(2,0) = -2\ , U(3,0) = 1, U(4,0) = 1, \ldots \qquad (14)$$

By using (14) into (13), the values of U(k, h) are obtained by recursive method as given below:

U (0,1) = -6; U (1, 1) = 0, U (2, 1) = 18, U (3, 1) = -24, U (4, 1) = 0, U (5, 1) = 36, U (6, 1) = 42, U (0, 2) = 36, U (1, 2) = -36, U (2, 2) = -108; U (3, 2) = 288, U (4, 2) = -108, U (5, 2) = -324, U (6, 2) = 756, U (7, 2) = -432

The above values of $U(k,h\ )$ are substituted into (8). Solution for the series is obtained as:

$$u(x,t) = \{1 + x - 2x^2 + x^3 + x^4 - 2x^5 + x^6 + x^7 - 2x^8 + x^9 + \cdots\} + \{-6t + 18x^2t - 24x^3t + 36x^5t - 42x^6t + \cdots\} + \{36t^2 - 36xxt^2 - 108x^2xt^2 + 288x^3t^2 - 180x^4t^2 - 324x^5t^2 + 756x^6t^2 - \cdots\} + \cdots$$

The above equation is written as:

$$u(x,t) = \frac{1+2x}{x^2+x+1} + \frac{-6(1+2x)}{(x^2+x+1)^2}t + \frac{36(1+2x)}{(x^2+x+1)^3}t^3 + \cdots \qquad (15)$$

The analytical solution of $u(x,t)$ is:

$$u(x,t) = \frac{1+2x}{x^2+x+6t+1}$$

Given below is a comparison of the approximate and exact solution obtained by using DTM:

x	*t*	*approximate*	*exact*
0.01	0.01	0.953182651	0.953181946
0.01	0.03	0.857224848	0.857070834
0.01	0.05	0.780365725	0.778566522
0.02	0.01	0.962607119	1.019208154
0.02	0.03	0.866525859	0.866377874
0.02	0.05	0.79811573	0.796453528

4. CONCLUSIONS

The paper reviewed DFT and the areas of their application. The theory and method of analysis was first examined, and various equations, forms, and methods of solving non-linear heat equations were examined. The method was then applied to an example with two variables and by considering certain initial boundary conditions. Various equations were derived, and a comparison of values for (x, t) was obtained for approximate and exact solutions with DTM. The differences in the values for the methods are very small. The conclusion drawn is that DTM is a stable and powerful method to solve a wide range of problems, and it helps to quickly develop rapid convergence for different solutions and to solve non-linear heat equations.

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